Calculate the limit

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$$\lim_{n\to\infty}\sqrt[n]{\frac{1}{4}\cdot\frac{3}{8}\cdots\frac{2n-1}{4n}}$$

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Note that $\sqrt[n]{\frac{1}{4}\cdot\frac{3}{8}\cdots\frac{2n-1}{4n}} = \sqrt[n]{\frac{1}{2^n}\cdot\frac{(2n-1)!!}{(2n)!!}} = \frac{1}{2}\sqrt[n]{P_n}$, where $P_n := \frac{(2n-1)!!}{(2n)!!}$.

Since $P_n^2 < \frac{(2n-1)!!}{(2n)!!}\cdot\frac{(2n)!!}{(2n+1)!!} = \frac{1}{2n+1} \Leftrightarrow P_n < \frac{1}{\sqrt{2n+1}}$,

$P_n > \frac{(2n-1)!!}{(2n+1)!!} = \frac{1}{2n+1}$ and $\lim_{n\to\infty}\sqrt[n]{\frac{1}{2n+1}} = \lim_{n\to\infty}\sqrt[n]{\frac{1}{\sqrt{2n+1}}} = 1$ then by

Squeeze Principle $\lim_{n\to\infty}\sqrt[n]{P_n} = 1$ and, therefore, $\lim_{n\to\infty}\sqrt[n]{\frac{1}{4}\cdot\frac{3}{8}\cdots\frac{2n-1}{4n}} = \frac{1}{2}$.